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LETTER TO THE EDITOR

Stopping a slow-light soliton: an exact solution

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Abstract
We investigate propagation of a slow-light soliton in Λ-type media such as atomic vapours and Bose–Einstein condensates. We show that the group velocity of the soliton monotonically decreases with the intensity of the controlling laser field, which decays exponentially after the laser is switched off. The shock wave of the vanishing controlling field overtakes the slow soliton and stops it, while the optical information is recorded in the medium in the form of spatially localized polarization. In the strongly nonlinear regime we find an explicit exact solution describing the whole process.

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In this letter we study the interaction of light with a gaseous active medium whose working energy levels are well approximated by the Λ-scheme. The model is a very close prototype for a gas or Bose–Einstein condensate (BEC) of alkali atoms, whose interaction with light is approximated by the structure of levels of the Λ-type. Reference [1] uses this model to develop the theory of electromagnetically induced transparency (EIT). Typically, in experiments [2–7] the pulses have the length of microseconds, which is much shorter than the coherence lifetime and longer than the optical relaxation times. We study the problem in the approximation that the atoms are cooled down to microkelvin temperatures in order to suppress the Doppler shift and increase the coherence lifetime for the ground levels. The medium is illuminated by two circularly polarized optical beams co-propagating in the z-direction. One σ^--polarized field is denoted as a, and the other σ^+-polarized field is denoted as b. We introduce two corresponding Rabi frequencies

$$\Omega_a = \frac{2\mu_a\mathcal{E}_a}{\hbar}, \qquad \Omega_b = \frac{2\mu_b\mathcal{E}_b}{\hbar}, \tag{1}$$

and study dynamics of the fields within the slowly varying amplitude and phase approximation (SVEPA). Here $\mu_{a,b}$ are dipole moments of corresponding transitions in the atom, and $\mathcal{E}_{a,b}$ are the field amplitudes.

In the interaction picture and within the SVEPA the Hamiltonian $H_\Lambda = H_0 + H_I$ describing the interaction of a three-level atom with the fields is defined as follows,

$$H_0 = -\frac{\Delta}{2}D, \qquad H_I = -\frac{1}{2}(\Omega_a|3\rangle\langle 1| + \Omega_b|3\rangle\langle 2|) + \text{h.c.}, \tag{2}$$

where $D = I - 2|3\rangle\langle 3|$ is a 3×3 diagonal matrix. We set $\hbar = 1$. Dynamics of the fields is described by the Maxwell equations, which within the SVEPA take the form

$$\partial_\zeta H_I = \mathrm{i}\frac{\nu_0}{4}\,[D, \rho]\,. \tag{3}$$

Here we have introduced new variables $\zeta = (z - z_0)/c$, $\tau = t - (z - z_0)/c$, ρ is the density matrix in the interaction representation, n_A is the density of atoms and ϵ_0 is the vacuum susceptibility. For many experimental situations it is typical that the coupling constants $\nu_{a,b} = (n_A|\mu_{a,b}|^2\omega_{a,b})/\epsilon_0$ are almost the same. Therefore we assume that $\nu_a \approx \nu_b = \nu_0$. Together with the Liouville equation,

$$\partial_\tau\rho = -\mathrm{i}[H_I, \rho], \tag{4}$$

we obtain a system of equations (3), (4), which is exactly solvable in the framework of the inverse scattering (IS) method [8–11]. Below we explain why for our solutions the influence of relaxation is negligible. Therefore the relaxations terms are not included in equation (4).

We assume that before arrival of the soliton the atom–field system is prepared in a state corresponding to a typical experimental setup (see e.g. [2, 3, 5]):

$$\Omega_a^{(0)} = 0, \qquad \Omega_b^{(0)} = \Omega(\tau), \qquad |\psi_{\text{at}}\rangle = \mathrm{e}^{-\mathrm{i}\frac{\Delta}{2}\tau}|1\rangle. \tag{5}$$

The state satisfies the Maxwell–Bloch system of equations (3), (4). The field $\Omega(\tau)$ plays the role of the controlling background field generated by a laser. In [11] for the constant background field $\Omega(\tau) = \Omega_0 = \Omega_0^*$ we found the slow-light soliton

$$\Omega_a = \frac{-\mathrm{i}\sqrt{2\varepsilon_0}\Omega_0}{\sqrt{\varepsilon_0 + \sqrt{\varepsilon_0^2 - \Omega_0^2}}}\operatorname{sech}\phi_s, \qquad \Omega_b = \Omega_0\tanh\phi_s, \tag{6}$$

and its atomic counterpart. Here $\phi_s = \frac{\nu_0\zeta}{2\varepsilon_0} - \frac{\tau}{2}\left(\varepsilon_0 - \sqrt{\varepsilon_0^2 - \Omega_0^2}\right) + \phi_0$. This type of solutions for constant background field was also discussed in [9]; approximative descriptions of pulse propagation based on the concept of effective time can be found in [5, 12, 14, 15]. In equation (6), for simplicity we set $\Delta = 0, \varepsilon_0 > \Omega_0$. The meaning of the parameter ε_0 is explained below. It can be readily seen that the speed of the slow-light soliton sent into the system depends on the intensity of the controlling background field. In the simplifying approximation $\frac{\Omega_0^2}{\varepsilon_0^2} \ll 1$, the group velocity reads $v_g \approx c\frac{\Omega_0^2}{2\nu_0}$ [11]. This expression immediately suggests a plausible conjecture that when the controlling field is switched off the soliton stops propagating while the information borne by the soliton remains in the medium in the form of an imprinted polarization flip.

In this letter we provide analytical solution substantiating the dynamical mechanism of the nonlinear control formulated above. We envisage the following dynamics scenario. We assume that the slow-light soliton is propagating on the background of the constant controlling field Ω_0 according to the exact solution [11]. Suppose that at the moment in time $t = 0$ the laser source of the controlling field is switched off. We assume that after this moment the background field will exponentially decay with some characteristic rate α. The exponential front of the vanishing controlling field will then propagate into the medium, starting from the point $z = z_0$, where the laser is placed. The state of the quantum system equation (5) is dark for the controlling field. Therefore the medium is transparent for the spreading front of the vanishing field, which then propagates with the speed of light, eventually overtaking the slow-light soliton and stopping it.

To realize the mechanism described above we define the time dependence of $\Omega(\tau)$ as follows:

$$\Omega(\tau) = \begin{cases} \Omega_0, & \tau < 0 \\ \Omega_0 \exp(-\alpha\tau), & \tau \geqslant 0. \end{cases} \tag{7}$$

This regime of switching the field off is quite realistic. An experimental setup, where the parameter α becomes experimentally adjustable, can be easily envisaged. Greater values of α correspond to steeper fronts of the incoming background wave. Developing further the Darboux–Bäcklund technique of our previous work [11] to treat the case of a time-dependent background field, we construct the exact analytical single-soliton solution of the time-dependent problem. For the fields the solutions are

$$\tilde{\Omega}_a = \frac{(\lambda^* - \lambda)w(\tau,\lambda)}{\sqrt{1 + |w(\tau,\lambda)|^2}} \mathrm{e}^{\mathrm{i}\tilde{\theta}_s} \operatorname{sech}\tilde{\phi}_s, \qquad \tilde{\Omega}_b = \frac{(\lambda - \lambda^*)w(\tau,\lambda)}{1 + |w(\tau,\lambda)|^2} \mathrm{e}^{\tilde{\phi}_s} \operatorname{sech}\tilde{\phi}_s - \Omega(\tau), \tag{8}$$

and for the atomic medium

$$|\tilde{\psi}_{\mathrm{at}}\rangle = \mathrm{e}^{-\mathrm{i}\frac{\Delta}{2}\tau}\left[\left(\frac{\lambda^* - \Delta}{|\lambda - \Delta|} - \frac{\tilde{\Omega}_a \exp\left(\mathrm{i}\left(\frac{\nu_0\zeta}{2(\lambda-\Delta)} - \tilde{\theta}_0\right) - z(\tau,\lambda)\right)}{2|\lambda - \Delta| w(\tau,\lambda)}\right)|1\rangle \right.$$
$$\left. + \frac{\tilde{\Omega}_a}{2|\lambda - \Delta| w(\tau,\lambda)}|2\rangle - \frac{\tilde{\Omega}_a}{2|\lambda - \Delta|}|3\rangle\right]. \tag{9}$$

The parameters of the slow-light soliton are

$$\tilde{\phi}_s = \tilde{\phi}_0 - \frac{\nu_0 \operatorname{Im}(\lambda)\zeta}{2|\Delta - \lambda|^2} + \operatorname{Re}(z(\tau,\lambda)) + \frac{1}{2}\ln(1 + |w(\tau,\lambda)|^2),$$
$$\tilde{\theta}_s = \tilde{\theta}_0 - \frac{\nu_0\zeta}{2}\operatorname{Re}\left(\frac{1}{\lambda - \Delta}\right) + \operatorname{Im}(z(\tau,\lambda)), \qquad \lambda \in \mathrm{C}. \tag{10}$$

We find the functions w, z. For $\tau < 0$ these functions are

$$w \equiv w_0 = \frac{\Omega_0}{\lambda + \sqrt{\lambda^2 + \Omega_0^2}}, \qquad z(\tau,\lambda) = \frac{\mathrm{i}}{2}\Omega_0 w_0 \tau, \tag{11}$$

while for $\tau \geqslant 0$ the functions w, z take the form

$$z(\tau,\lambda) = -\alpha\gamma\tau + \ln\frac{\mathcal{C}J_{-\gamma}\left(-\frac{\Omega(\tau)}{2\alpha}\right) + J_\gamma\left(-\frac{\Omega(\tau)}{2\alpha}\right)}{\mathcal{C}J_{-\gamma}\left(-\frac{\Omega_0}{2\alpha}\right) + J_\gamma\left(-\frac{\Omega_0}{2\alpha}\right)}, \tag{12}$$

$$w(\tau,\lambda) = \mathrm{i}\frac{\mathcal{C}J_{1-\gamma}\left(-\frac{\Omega(\tau)}{2\alpha}\right) - J_{\gamma-1}\left(-\frac{\Omega(\tau)}{2\alpha}\right)}{\mathcal{C}J_{-\gamma}\left(-\frac{\Omega(\tau)}{2\alpha}\right) + J_\gamma\left(-\frac{\Omega(\tau)}{2\alpha}\right)}, \tag{13}$$

where $\gamma = \frac{\alpha + \mathrm{i}\lambda}{2\alpha}$ and J_ν are Bessel functions. The constant $\mathcal{C}$ is uniquely defined by the condition $w(0,\lambda) = w_0$,

$$\mathcal{C} = \frac{-\mathrm{i}w_0 J_\gamma\left(-\frac{\Omega_0}{2\alpha}\right) + J_{\gamma-1}\left(-\frac{\Omega_0}{2\alpha}\right)}{J_{1-\gamma}\left(-\frac{\Omega_0}{2\alpha}\right) + \mathrm{i}w_0 J_{-\gamma}\left(-\frac{\Omega_0}{2\alpha}\right)}. \tag{14}$$

To identify a physically relevant solution we require that $w(\infty,\lambda) = 0$. This requirement places a restriction on the parameter λ, such that $\operatorname{Im}(\lambda) < 0$ and hence $\operatorname{Re}(\gamma) > 1/2$.

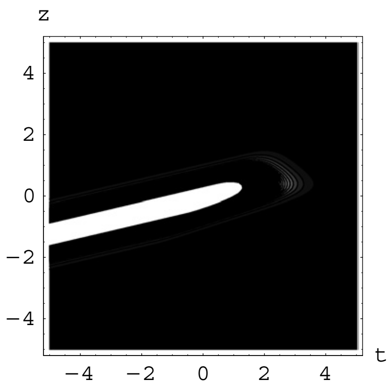

Figure 1. The slow-light soliton is stopped [5]. Contour plot of the intensity I_a of the field $\tilde{\Omega}_a$. The parameters used for all the plots in this letter are the same: $\Delta = 0$, $\Omega_0 = 2$, $\varepsilon_0 = 2.1$, $\nu_0 = 10$, $\alpha = 1$.

It is easy to show that the group velocity of the slow-light soliton reads

$$\frac{v_g}{c} = \frac{|w(\tau,\lambda)|^2}{\frac{\nu_0(1+|w(\tau,\lambda)|^2)}{2|\Delta-\lambda|^2} + |w(\tau,\lambda)|^2}. \qquad (15)$$

Note that in the case of the constant background field, i.e. in the case $\alpha = 0$, the conventional expressions for the slow-light soliton (6) along with the expression for the group velocity—the main motivational quantity for this letter—can be readily recovered from equations (8), (15).

We also calculate the distance $\mathcal{L}_s(\alpha)$ that the slow-light soliton will propagate from the moment $t = 0$, when the laser is switched off, until the signal is fully stopped. This distance is

$$\mathcal{L}_s(\alpha) = \frac{2c|\Delta-\lambda|^2}{\nu_0\,\mathrm{Im}(\lambda)}\tilde{\phi}_s|_{\tau=0}^{\tau=\infty} = \frac{2c|\Delta-\lambda|^2}{\nu_0|\mathrm{Im}(\lambda)|}\left[\ln\sqrt{1+|w_0|^2} - \mathrm{Re}(z(\infty,\lambda))\right]. \qquad (16)$$

It is clear that

$$z(\infty,\lambda) = \ln\frac{\mathcal{C}\left(-\frac{\Omega_0}{4\alpha}\right)^{-\gamma}\Big/\,\Gamma(1-\gamma)}{\mathcal{C}J_{-\gamma}\left(-\frac{\Omega_0}{2\alpha}\right) + J_{\gamma}\left(-\frac{\Omega_0}{2\alpha}\right)}.$$

Note that $\mathrm{Re}\,z(\infty,\lambda) \leqslant 0$ and hence $\mathcal{L}_s(\alpha) > 0$. The case when the controlling field is instantly switched off corresponds to the limit $\alpha \to \infty$. In this limit the profile of the background field approaches the Heaviside step-function, and we find that $\lim_{\alpha\to\infty}\mathrm{Re}\,z(\infty,\lambda) = 0$. In this case, as is intuitively evident, the soliton will still propagate over some finite distance. Note that the limits $\tau \to \infty$ and $\alpha \to \infty$ do not commute and the latter should be taken after the former.

In what follows we assume the parameter λ to be imaginary, i.e. $\lambda = -\mathrm{i}\varepsilon_0$, with $\varepsilon_0 > \Omega_0$. We demonstrate the slow-light dynamics in figure 1, where we show how the slow-light soliton stops and disappears. In figure 2 we demonstrate the behaviour of the intensity in the channel b. The groove in the constant background field corresponds to the slow-light soliton. This groove is complementary to the peak in the channel a. The shock wave, whose front has an exponential profile, propagates with the speed of light, reaches the slow-light soliton and stops it. Note that after the collision with the slow-light soliton the front of the shock wave shows some short peak at a level higher than the background intensity. This effect reflects an essentially nonlinear nature of interactions inherent to the considered dynamical system. In

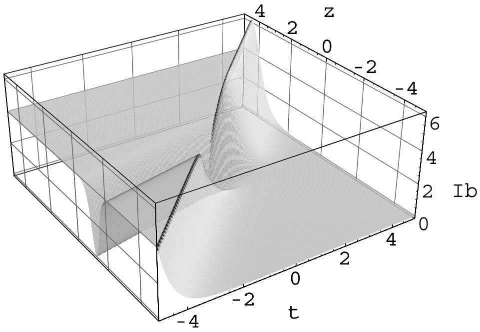

Figure 2. The intensity I_b of the field $\tilde{\Omega}_b$ as a function of time t and space z. The parameters are defined in figure 1. The groove corresponds to the slow-light soliton. The soliton collides with the shock wave of the vanishing control field, whose front has an exponential profile.

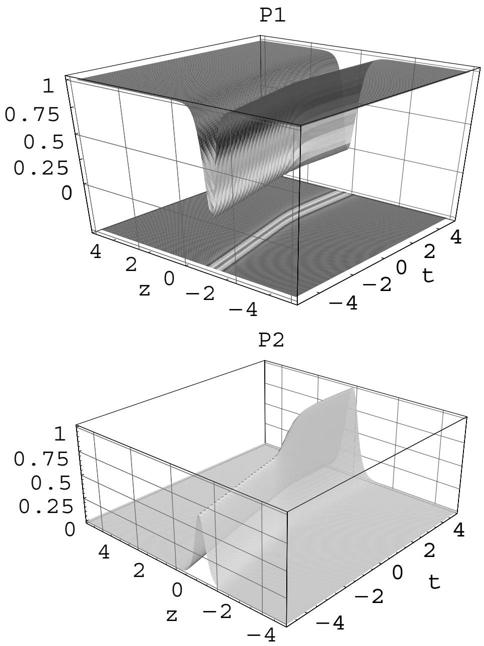

Figure 3. Imprinting the information. We plot the dynamics of the population P_1 of the level $|1\rangle$ and P_2 of the level $|2\rangle$ as functions of time t and space z. It is clear that after the collision the populations freeze.

figure 3 we plot the dynamics of the polarization flip in the atomic medium, which bears the information and stores it in the medium. Before the shock wave has approached, the spatially localized population of the level $|2\rangle$ moves together with the slow-light soliton as a composite whole. This is a nonlinear analogue of the dark-state polariton [13]. After the shock wave has arrived the population is frozen at the position where it was hit by the shock wave.

It is important to note that, before the slow-light soliton is stopped, the population on the level $|2\rangle$ does not reach unity, because there is a remnant population in the upper state $|3\rangle$. However, after the polariton is completely stopped, the level $|3\rangle$ depopulates, while the population of the level $|2\rangle$ reaches unity at the maximum of the signal. Before and after the collision, the population of the level $|1\rangle$ at the minimum of the groove always vanishes. The result is valid for zero detuning. From our solution (9) it is not difficult to estimate the maximum population of the level $|2\rangle$ for finite detuning after the soliton is completely stopped: $\varepsilon_0^2 \big/ \left(\varepsilon_0^2 + \Delta^2\right)$. Any destructive influence of the relaxation processes on the overall picture of dynamics described above is negligible. Indeed, as we show in this letter (see also [11]), the population of the upper level $|3\rangle$ is proportional to the intensity of the background field Ω_0, which is required to be small, to ensure a small velocity of the signal. Numerical analysis shows that analytical solutions given above are stable with respect to relaxation from the level $|3\rangle$. This result concurs with general numerical treatments of storing optical information reported in [7, 12].

Discussion. In this letter we have investigated the dynamics of a slow-light soliton whose group velocity explicitly depends on the background (controlling) field. Taking advantage of an explicit exact solvable example, we demonstrate that the soliton can indeed be stopped, provided that the background field vanishes. After the signal is stopped, the information borne by the soliton is imprinted into the medium in the form of a spatially localized polarization flip. The position of the spatially localized optical memory imprint is controlled by the experimentally adjustable parameter α. Our approach allows addressing of optical memory recorded into an atomic medium at an exact location without changing the characteristic size of the spatial domain occupied by the memory. The imprinted memory can be subsequently read.

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